Possible Implications of the Atmospheric, the Bugey, and the Los Alamos Neutrino Experiments [*]

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Abstract

A combined analysis of the terrestrial neutrino experiments and the Kamiokande observation of atmospheric neutrino anomaly is performed under the assumption of the existence of dark-matter-mass neutrinos, as suggested by the recent Los Alamos experiment. In the three-flavor mixing scheme of neutrinos it is shown that the constraints from these experiments are so strong that the patterns of mass hierarchy and flavor mixing of neutrinos are determined almost uniquely depending upon the interpretation of the atmospheric neutrino anomaly.

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There have been accumulating indirect evidences for nonvanishing masses and the flavor mixings of neutrinos. They include the solar neutrino deficit [1] which may be interpreted by either the Mikheyev-Smirnov-Wolfenstein mechanism [2] or the vacuum neutrino oscillation [3], both being based upon the notion of flavor mixing. The second in the list is the atmospheric neutrino anomaly first observed by the Kamiokande experiment [4] and subsequently confirmed by other detectors [5,6], which strongly indicates the large-angle flavor mixing of neutrinos.

The recent announcement of the discovery of nonzero neutrino mass by the Liquid Scintillator Neutrino Detector (LSND) experiment [7,8] in Los Alamos may have brought the first direct evidence for the neutrino masses and the flavor mixing. The experiment may have observed the neutrino oscillation $\bar{\nu}_\mu \to \bar{\nu}_e$ with oscillation parameters $\Delta m^2 \simeq 1 - 12$ eV^2 and $\sin^2 2\theta \simeq 10^{-2} - 10^{-2.5}$, if interpreted by the two-flavor mixing scheme. The result may be marginally compatible with the earlier results obtained by the Los Alamos [9] and the BNL experiments [10] and by the KARMEN collaboration experiment [11]. Clearly the result, if confirmed by the continuing runs, has tremendous implications to particle physics and cosmology [12–14].

In this paper we try to extract the implications of the possible existence of the dark-matter-mass neutrinos, as suggested by the LSND result, in the light of the experimental informations from the underground, the reactor and the accelerator experiments. We first observe, as many authors do [12–14], that one cannot explain the above three phenomena simultaneously by the three-flavor mixing scheme without introducing sterile neutrinos. It is simply due to the fact that the three-flavor scheme cannot accommodate three hierarchically different mass scales, $\Delta m^2 \simeq (1 - 12)$eV^2 for LSND, $\Delta m^2 \simeq 10^{-2}$eV^2 for the atmospheric neutrino anomaly, and $\Delta m^2 \simeq 10^{-6} - 10^{-5}$eV^2 ($\simeq 10^{-10}$eV^2) for the MSW (vacuum mixing) solution of the solar neutrino problem.

We derive the constraints imposed on neutrino masses and mixing angles via a combined analysis of the reactor and the accelerator data and the atmospheric neutrino anomaly under the assumption that at least one of the neutrinos has a mass which falls into the mass

range 1-10 eV which is appropriate for cosmological hot dark matter. This assumption will be referred to as the assumption of dark-matter-mass neutrinos (DMMN) hereafter. We employ the mixing scheme based on three-generation neutrinos, as beautifully confirmed by the LEP experiments [15]. It will be demonstrated that it is essential to use the three-flavor mixing scheme, rather than optional use of various two-flavor mixings, for drawing correct interpretation of the data. We will also consider the restrictions imposed by the neutrinoless double β decay [16].

Amazingly, the constraints imposed by a minimal set of data, the atmospheric and the Bugey [17] experiments, and the assumption of DMMN are so restrictive as to determine the masses and the mixing patterns of three flavor neutrinos. Only a few patterns are allowed: (A) light "ν_e" and almost degenerate strongly mixed heavy "ν_μ" and "ν_τ", and its mass-inverted version, or (B) light "ν_τ" and almost degenerate strongly mixed heavy "ν_e" and "ν_μ", and its mass-inverted one. The choice of the solutions (A) or (B) is dictated by the interpretation of the atmospheric neutrino anomaly. The pattern (A) follows if we interpret the atmospheric neutrino anomaly as due to the $\nu_\mu \to \nu_\tau$ oscillation, while (B) results if it is due to the $\nu_\mu \to \nu_e$ oscillation.

We stress that neither combinations with atmospheric nor with solar neutrino data are compelling. The interpretation is somewhat more involved in the latter case because there still exist three types of solutions to the solar neutrino problem based on the neutrino flavor mixing; the small and the large-angle MSW solutions in addition to the vacuum mixing one. The analysis of this combination will be presented elsewhere [18]. Nonetheless, we should mention that our theoretical prejudice prefers the case with atmospheric neutrino data over the other one. It is natural to introduce a sterile neutrinos to accommodate the third experimental data left over in both cases. It is, however, difficult to explain the atmospheric neutrino anomaly by introducing the mixing with sterile neutrinos. In doing so one encounters the trouble with the light-element nucleosynthesis [19].

We make use of one crucial aspect of the atmospheric neutrino data in our analysis. Namely, the Kamiokande group recently provided a new data set called the multi-GeV sam-

ple [20]. They consist of the events with higher energy, $\gtrsim 1.33$ GeV, than the previously reported data. The important feature of the new data is that, because of the higher energy, the path-length dependence of the oscillation probability can be probed by measuring the zenith-angle dependence. It is striking that it can be perfectly fitted by the neutrino oscillation with mixing parameters $\Delta m^2 \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta \simeq 1$ [20]. Such quantitative agreement with the zenith-angle dependence is the strongest support for the neutrino oscillation interpretation of the atmospheric neutrino anomaly.

We now make three basic observations in view of the data of the LSND and the Kamiokande atmospheric neutrino experiments. (1) To have gross deficit in the ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ we need at least one large mixing angle. (2) To be consistent with the rate of the oscillation events of the order of $\sim 5 \times 10^{-3}$ or less at least one mixing angle has to be small. (3) The feature of the atmospheric neutrino data indicates that the one of the three Δm_{ij}^2 is of the order of $\simeq 10^{-2}$eV^2. Notice that we cannot have two Δm_{ij}^2 of the order of 10^{-2}eV^2 because it contradicts with the assumption of DMMN, $\Delta m^2 \gtrsim 1$eV^2.

Based on these observations we classify the hierarchy of the neutrino masses into the following two types:

$$\mathrm{a}: m_3^2 \approx m_2^2 \gg m_1^2 \qquad\qquad \mathrm{b}: m_1^2 \gg m_2^2 \approx m_3^2 \qquad\qquad (1)$$

Here the symbols $\approx$ and $\gg$ imply the differences by $\sim 10^{-2}$eV^2 and $\sim 1 - 100$eV^2, respectively. Throughout the analysis in this paper the relative magnitude of the masses connected by $\approx$ does not matter. The other types of mass hierarchies which are obtained by permuting 1, 2, and 3 will automatically be taken care of because they merely represent relabeling the mass eigenstates.

We recollect the basic formula of the oscillation probabilities with three flavors of neutrinos. We introduce the neutrino mixing matrix U which relates the flavor- and the mass-eigenstates as $\nu_\alpha = U_{\alpha i}\nu_i$, where the flavor index α runs over e, μ and τ and the mass-eigenstate index i runs over 1 to 3. We assume the CP invariance in the present analysis. In this case the mixing matrix U is real and contains only three angles θ_{ij}.

With the mixing matrix the oscillation probability of neutrinos of energy E after traversing the distance L can be written as

$$P(\nu_\beta \rightarrow \nu_\alpha) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$$

$$= \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right), \tag{2}$$

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$. As a convenient parametrization of the matrix U we use the so called standard form of the Kobayashi-Maskawa matrix, which is now adopted for the neutrino mixing matrix:

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix}, \tag{3}$$

where c_{ij} and s_{ij} are the short-hand notations for $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. We note that the three real angles can all be made to lie in the first quadrant by an appropriate redefinition of neutrino phases.

The expressions of the oscillation probability are rather cumbersome involving many angle factors. Therefore, we shall derive the approximate formulas by taking into account the mass hierarchies and the experimental parameters of the three experiments. The oscillation probability which corresponds to the LSND experiment is approximately given by

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 4c_{12}^2 c_{13}^2 (s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \sin^2\left(\frac{\Delta m_{12}^2}{4E}L\right). \tag{4}$$

where two terms with $\Delta m_{12}^2 \approx \Delta m_{13}^2$ (which differ only by $10^{-2}\mathrm{eV}^2$) are combined and the term with Δm_{23}^2 is ignored. The former procedure can be neatly done by utilizing the orthogonality relation of the mixing matrix. The latter approximation is completely legitimate because the term is smaller than others by factor $10^{-4} - 10^{-8}$ owing to the mass hierarchy $\Delta m_{23}^2/\Delta m_{12}^2 \simeq 10^{-2} - 10^{-4}$.

If the atmospheric neutrino anomaly is attributed to the $\nu_\mu \rightarrow \nu_\tau$ oscillation the relevant formula is

$$P(\nu_\mu \to \nu_\tau) = 2(s_{12}c_{23} + c_{12}s_{23}s_{13})^2(s_{12}s_{23} - c_{12}c_{23}s_{13})^2$$

$$+ 4c_{23}s_{23}c_{13}^2(c_{12}c_{23} - s_{12}s_{23}s_{13})(c_{12}c_{23} + s_{12}c_{23}s_{13})\sin^2(\frac{\Delta m_{23}^2 L}{4E}). \quad (5)$$

In (5) the sine-squared factors with large Δm^2 of $\gtrsim 1\text{eV}^2$ are replaced by the average value $\frac{1}{2}$, which can be justified because of the rapid oscillations; the argument of the sine is $\sim 10 - 10^3(10^4 - 10^6)$ for L= $10(10^4)$ Km for $\Delta m^2 = 1 - 100\text{eV}^2$ and E= 1GeV

We note that there exist the possibility that the atmospheric neutrino anomaly is due to the $\nu_\mu \to \nu_e$ oscillation, the possibility one might not naively expect. It is perfectly consistent with a small event rate in the LSND experiment because the relevant scales of path length and neutrino energy involved in these two experiments are much different. In this case the formula for the oscillation probability to be used is

$$P(\nu_\mu \to \nu_e) = 2c_{12}^2 c_{13}^2(s_{12}c_{23} + c_{12}s_{23}s_{13})^2$$

$$- 4s_{12}s_{23}c_{13}^2 s_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13})\sin^2(\frac{\Delta m_{23}^2 L}{4E}). \quad (6)$$

Finally the formula for the Bugey experiment takes the form

$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = 2c_{12}^2 c_{13}^2(1 - c_{12}^2 c_{13}^2)$$

$$+ 4s_{12}^2 c_{13}^2 s_{13}^2 \sin^2(\frac{\Delta m_{23}^2 L}{4E}), \quad (7)$$

where the terms with Δm_{12}^2 are averaged as before. It can be justified because the argument of the sine term is of the order of $10 - 10^3$ with $\Delta m^2 = 1 - 100\text{eV}^2$, E= 4MeV, and L= 40m, the typical parameters of the Bugey experiment. The second term of (7) may be neglected (as we will do) because sine-squared factor is $\sim 10^{-2}$ for $\Delta m^2 = 10^{-2}\text{eV}^2$.

We first examine the case that the atmospheric neutrino anomaly is attributed to the $\nu_\mu \to \nu_\tau$ oscillations. Our discussion does not distinguish the types a and b until we address the constraint due to the double β decay.

We demand, for consistency with the gross features of the LSND, the Bugey, and the atmospheric neutrino experiments, the following constraints:

$$c_{12}^2 c_{13}^2(s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \equiv \epsilon \lesssim 10^{-3} \quad (8)$$

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$$c_{12}^2 c_{13}^2 (s_{12}^2 c_{13}^2 + s_{13}^2) \lesssim \delta = 2.5 \times 10^{-2} \tag{9}$$

$$(s_{12}c_{23} + c_{12}s_{23}s_{13})^2 (s_{12}s_{23} - c_{12}c_{23}s_{13})^2 \leq 0.1, \tag{10}$$

$$4c_{23}s_{23}c_{13}^2 (c_{12}c_{23} - s_{12}s_{23}s_{13})(c_{12}s_{23} + s_{12}c_{23}s_{13}) \simeq 1. \tag{11}$$

The constraints (8) comes from the LSND experiment. We treat ϵ as a small number of the order of $\sim 10^{-3}$ or less. Our discussion will be insensitive to the number and we use it as a tentative guide when we address the consistency with other experiments. The equation (9) is due to the bound $1 - P(\bar{\nu}_e \to \bar{\nu}_e) \lesssim 5\,\%$ obtained in the Bugey experiment [17]. It includes statistical and systematic uncertainties. The remaining two restrictions are from the features of the atmospheric neutrino data that the zenith-angle dependence is well described by an effective two-flavor-mixing ansatz with $\Delta m^2 \simeq 10^{-2} \mathrm{eV}^2$ and $\sin^2 2\theta \simeq 1$. The constraint (10) arises from a mild requirement that the first term of (5) should be less than 0.2 so as not to disturb the effective two-flavor description. We emphasize that the constraints from the atmospheric neutrino data take the simple forms (10) and (11) because of the mass hierarchy $\Delta m_{12}^2 \approx \Delta m_{13}^2 \gg \Delta m_{23}^2$.

We first notice that from the Bugey constraint(9) that $X \equiv c_{12}^2 c_{13}^2$ must satisfy the inequality $X^2 - X + \delta \geq 0$. This inequality is so powerful that restricts the value of X into the two tiny regions $0 \leq X \leq \delta$ and $1 - \delta \leq X \leq 1$. On the other hand, we must have $c_{13}^2 \simeq O(1)$ in order to satisfy the requirement (11). Thus, we have either $c_{12}^2 \simeq \delta$ or $c_{12}^2 \simeq 1$ corresponding to the small-X and the large-X solutions, respectively. It is also required that $c_{23}s_{23} \simeq \frac{1}{2}$ in order to maximize (11). The small-X solution is then inconsistent with (11). We end up with the unique solution

$$(\mathrm{A}) \qquad s_{12}^2 \approx s_{13}^2 \simeq \epsilon, \qquad c_{23}^2 \approx s_{23}^2 \simeq \frac{1}{2} \tag{12}$$

where we have also utilized the LSND constraint (8) to push $s_{12}^2 \simeq \delta$ down to $s_{12}^2 \simeq \epsilon \lesssim 10^{-3}$.

We have explicitly verified that the allowed mixing pattern implied by (12) is physically unique throughout the varying mass hierarchies obtained by the cyclic permutations of 1-3

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of (1), as it should be. Namely, the light "ν_e" and the almost degenerate strongly mixed heavy "ν_μ" and "ν_τ" for the type-a, and the heavy "ν_e" and the almost degenerate strongly mixed light "ν_μ" and "ν_τ" for the type-b cases.

We now turn to the case that the atmospheric neutrino anomaly is caused by the $\nu_\mu \to \nu_e$ oscillation. In this case we replace the requirements (10) and (11) by

$$c_{12}^2 c_{13}^2 (s_{12} c_{23} + c_{12} s_{23} s_{13})^2 \leq 0.1, \tag{13}$$

and

$$-4 s_{12} s_{23} c_{13}^2 s_{13} (c_{12} c_{23} - s_{12} s_{23} s_{13}) \simeq 1, \tag{14}$$

respectively. By the similar procedure one can show that the consistent solution of the requirements (8), (9), (13), and (14) is uniquely given by

$$\text{(B)} \qquad c_{12}^2 \approx c_{23}^2 \simeq \sqrt{\epsilon}, \qquad c_{13}^2 \approx s_{13}^2 \simeq \frac{1}{2}. \tag{15}$$

The solutions of the other type of mass hierarchies can be obtained by the similar manner and correspond to the redefinition of the mass eigenstates. The allowed mixing pattern is again physically unique: The light "ν_τ" and the almost degenerate strongly mixed heavy "ν_e" and "ν_μ" for the type-a, and the heavy "ν_τ" and the almost degenerate strongly mixed light "ν_e" and "ν_μ" for the type-b mass hierarchies.

We note that the solutions (A) and (B) are subject to the additional constraints from other terrestrial experiments. While the solution (A) solves them automatically the nontrivial constraints arise for (B). In particular, the most stringent one comes from the Fermilab E531 experiment [23] for $\Delta m^2 \gtrsim 3 \text{eV}^2$ and the ν_μ disappearance experiment by the CDHS group [24] for $\Delta m^2 \lesssim 3 \text{eV}^2$. If we take the rate of the appearance events reported in [25] at its face value the solution (B) may be excluded apart from tiny regions. To establish the rate, however, an additional run of the experiment as well as its careful analysis would be required.

In the case of Majorana neutrinos a further constraint emerges from the non-observation of the neutrinoless double β decay. The quantity

$$< m_{\nu e} >= \sum_{j=1}^{3} \eta_j |U_{ej}|^2 m_j \qquad (16)$$

is constrained to be less than ∼1eV by the experiments [16] where $\eta_j = \pm 1$ is the CP phase. Notice that we are working with the representation in which the mixing matrix is real under the assumption of CP invariance.

Generally speaking, the constraint from the double β decay distinguishes between the type-a and the type-b mass hierarchies. In the type-a case there is a chance for cancellation between nearly degenerate two masses, but no chance in the type-b case because the heavy mass is carried by a unique mass eigenstate.

New features, however, arise in our consistent solutions obtained above. We first discuss the case of atmospheric neutrino anomaly due to the $\nu_\mu \rightarrow \nu_\tau$ oscillation. It can be shown that in the type-a mass pattern the double β constraint is automatically satisfied because the heavy masses are always multipled by small angle factors. On the contrary, the angle factors in front of the unique heavy mass are always of the order of unity in the type-b mass hierarchy. Therefore, there is no consistent solution of the double β-decay constraint for Majorana neutrinos in the type-b hierarchy.

In the case of atmospheric neutrino anomaly due to the $\nu_\mu \rightarrow \nu_e$ oscillation, the situation is somewhat different. In the type-a mass pattern there is a trouble because almost degenerate heavy masses are multiplied by $O(1)$ coefficients and a tuning, i.e., $s_{12}^2 c_{13}^2 = s_{13}^2$ to better than 0.1, is required for cancellation in addition to the requirement of opposite CP parities. On the contrary, in the type-b hierarchy, there is no trouble with the double β-decay constraint because the heavy mass is multiplied by small coefficients of the order of $\sqrt{\epsilon}$.

Thus, we have shown in this paper that the neutrino masses and the mixings are strongly constrained by the atmospheric and the terrestrial experiments under the assumption of DMMN suggested by the LSND experiments. The constraint is so severe that the mass and the mixing patterns are determined almost uniquely within the uncertainties of the neutrino types and the interpretations of the atmospheric neutrino anomaly. In the case of

the Majorana neutrinos the additional constraint from the double β decay selects out the unique natural solution in each interpretation.

Finally we give a few remarks.

(1) Our analysis in this paper is less powerful in constraining the absolute values of the neutrino masses than in restricting the relative masses and the mixing angles. All the constraints would be cleared by, for example, the type-a solution with $m_1 = 6$ eV, $m_2 = 6.5$ eV, and $m_3 = (6.5 + \epsilon)$ eV, which is also consistent with the direct measurements [21] and the cosmological considerations [22].

(2) We have not performed a full three-flavor analysis of the atmospheric neutrino anomaly but relied on the effective two-flavor interpretation of either $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_e$ channels. While the point deserves further study we are under a strong feeling that coexistence of the two channels does not spoil our solutions obtained in this paper. They certainly survive in the case of equal contributions of these two channels.

Note Added: After submitting the earlier version of this paper we became aware of two reports from the LSND group [25,26] with mutually conflicting conclusions.

REFERENCES

[1] B. T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) **38**, 47 (1995); K. S. Hirata et al., Phys. Rev. **D44**, 2241 (1991); Y. Suzuki, Nucl. Phys. B (Proc. Suppl.) **38**, 54 (1995); A. I. Abazov et al., Phys. Rev. Lett. **67**, 3332 (1991); J. N. Abdurashitov et al., Nucl. Phys. B (Proc. Suppl.) **38**, 60 (1995); P. Anselmann et al., Phys. Lett. **B285**, 376 (1992); ibid **327**, 377 (1994).

[2] S. P. Mikheyev and A. Smirnov, Nuovo Cim. **9C**, 17 (1986); L. Wolfenstein, Phys. Rev. **D17**, 2369 (1978).

[3] For recent analyses, V. Barger, R. J. N. Phillips and K. Whisnant, Phys. Rev. **D43**, 1110 (1991); P. I. Krastev and S. T. Petcov, Phys. Rev. Lett. **72**, 1960 (1994).

[4] K. S. Hirata et al., Phys. Lett. **B205**, 416 (1988); ibid **B280**, 146 (1992).

[5] D. Casper et al., Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy et al., Phys. Rev. **D46**, 3720 (1992).

[6] M. C. Goodman, Nucl. Phys. B (Proc. Suppl.) **38**, 337 (1995).

[7] New York Times, Tuesday 31, January 1995.

[8] W. C. Louis, Nucl. Phys. B (Proc. Suppl.) **38**, 229 (1995); D. O. Caldwell, XXXth Rencontres de Moriond, January 21-28, 1995, Villars, Switzerland.

[9] L. S. Durkin et al., Phys. Rev. Lett. **61**, 1811 (1988).

[10] L. Borodovsky et al., Phys. Rev. Lett. **68**, 274 (1992).

[11] B. Armbruster et al., Nucl. Phys. B (Proc. Suppl.) **38**, 235 (1995).

[12] J. R. Primack, J. Holtzman, A. Klypin, and D. O. Caldwell, Phys. Rev. Lett. **74**, 2160 (1995).

[13] G. M. Fuller, J. R. Primack, and Y. -Z. Qian, preprint astro-ph/9502081.

[14] G. Raffelt and J. Silk, preprint hep-th/9502306.

[15] The LEP Electroweak Working Group, LEPEWWG/94-01, cited in Review of Particle Properties, Phys. Rev. **D50**, 1416 (1994).

[16] M. K. Moe, Nucl. Phys. B (Proc. Suppl.) **38**, 36 (1995).

[17] B. Achkar et al., Nucl. Phys. **B434**, 503 (1995).

[18] H. Minakata, preprint TMUP-HEL-9503, hep-ph/9504222.

[19] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. **48**, 2543 (1993).

[20] Y. Fukuda et al., Phys. Lett. **B335**, 237 (1994).

[21] E. W. Otten, Nucl. Phys. B (Proc. Suppl.) **38**, 26 (1995).

[22] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Co., California, 1990).

[23] N. Ushida et al, Phys. Rev. Lett. **57**, 2897 (1986).

[24] F. Dydak et al., Phys. Lett. **134B**, 281 (1984).

[25] C. Athanassopoulos et al., preprint NUCLEX/9504002.

[26] J. E. Hill, preprint hep-ex/9504009.